

Mh4718 Worksheet 10

1. Solve the following IVPs by separation of variables. In each case check your solution against an approximate solution obtained using Euler's method:

(i) $\frac{dy}{dx} = 3x^2e^{-y}, y(0) = 1.$

(ii) $\frac{dy}{dx} = y + \frac{y}{x}, y(1) = 1.$

2. Find each fixed point for the following functions and determine whether fixed point iteration is locally convergent to it.

(i) $0.5x^2 + 0.5x$; (ii) $x^2 - 0.25x + 0.375$; (iii) $F(x) = \frac{9x - 20}{x}.$

3. If the iteration $x_{n+1} = \sqrt{10x_n - 9}, n = 0, 1, 2, \dots$ were to converge to a fixed point what are the possible values for that fixed point?

4. Solve the following system of linear equations:

$$\begin{aligned}4x + y + w &= 1 \\x + 4y + z + v &= 2 \\y + 4z + w &= -1 \\x + z + 4w + v &= 2 \\y + w + 4v &= 1\end{aligned}$$